

Portfolio optimization analysis of REITs in commercial real estate

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Abstract: With the rapid development of economy in China, the commercial real estate increasingly become the focus of many investors. It is more and more popular that REITs is applied in commercial real estate. How to promote REITs benefits in commercial real estate investment? We will discuss it in this paper. This paper is established on Markowitz portfolio, REITs model optimization model. Taking the commercial real estate sales price index in jan 2013-2018 April, we carry out an empirical research about the commercial real estate for REITs in portfolio optimization. Under the B coefficient of optimization model for measure Markowitz, we sought for maximum profit (or minimal risk) of commercial real estate projects investment portfolio.

1. Introduction

Real Estate Investment Trusts, referred to as REITs for short, is a kind of trust method that collects unspecified investors, collects funds, establishes some special fund or organization for real estate investment management, conducts real estate investment and management, and shares the real estate investment income together. REITs originated in the United States in the 1960s, and then gradually expanded to Europe, Australia, Japan, South Korea, Singapore and other places. Since the successful listing of China's first REITs (Guangzhou Urban Construction Yuexiu) in Hong Kong on December 21, 2005, the domestic real estate trust market continues to show a rapid development trend.

With the continuous emergence of macro-control policies for real estate in our country, the traditional financing channels (domestic loans, utilization of foreign capital, self-raised funds and other funds) are blocked, the financing conditions faced by real estate development enterprises are becoming more and more strict, and the pressure on the capital chain is becoming more and more urgent. Especially for commercial real estate, the demand for capital is large, and the problem of financing has become a big concern for major developers. China's commercial real estate industry is struggling on the road to sustainable development. The real estate industry has developed to a stage where it has to break through the original single financing channel and seek multi-channel financing. Since REITs are mainly used to invest in commercial real estate projects (including office buildings, shopping centers, restaurants, entertainment places, storage facilities, etc.). It is imperative to combine commercial real estate with REITs, which is also the long-term trend of domestic commercial real estate development and an effective way out for the normal and healthy development of Chinese commercial real estate.

However, how to improve the investment efficiency of REITs in commercial real estate projects nationwide? This is the main problem solved in this paper. This paper intends to use Markowitz model to establish REITs portfolio optimization model from the perspective of portfolio optimization. Based on the sales price index of commercial real estate from July 2013 to April 2018, this paper conducts an empirical study on REITs' portfolio optimization of commercial real estate in China. Under Markowitz optimization model measured by B coefficient, it seeks the portfolio of commercial real estate projects with the maximum return (or minimum risk), so as to promote the harmonious development of commercial real estate in China.

2. Model Setting and Empirical Analysis

In this paper, the risk-return is taken as the central variable in the process of establishing the

portfolio optimization model, thus determining the investment proportion of REITs in each item of the commercial real estate project portfolio and seeking the objective of obtaining the maximum return (or minimum risk) of the commercial real estate project portfolio.

2.1 Construction of Markowitz Optimization Model Measured by B Coefficient

In order to achieve the goal of REITs to obtain the maximum return (or minimum risk) in the commercial real estate project portfolio, this paper makes the following assumptions:

- (1) According to the statistical yearbook classification, commercial real estate projects are divided into: office buildings, commercial premises and others;
- (2) all kinds of commercial real estate investment projects meet the economic feasibility and technical feasibility;
- (3) do not consider various transaction costs;
- (4) The optimized investment combination is not restricted by capital resources;
- (5) The expected rate of return of various commercial real estate projects can be obtained through investigation and statistical analysis of data;
- (6) Optimizing the investment portfolio on the basis of comprehensively considering the expected return rate and risks.
- (7) The distribution of commercial real estate investment portfolio is normal.

The core of the modern portfolio theory developed by Markowitz et al. is to use the risks, expected returns and covariance between individual investments to determine an effective portfolio². According to Markowitz's theory, we assume that REITs have three different ways to invest in commercial real estate, namely, to invest in commercial premises, office buildings and other premises, of which commercial premises include shopping centers, restaurants and entertainment venues, and other commercial premises include storage facilities. The commercial real estate portfolio model based on Markowitz theory is obtained.

Objective Function: Maximize Income $MaxE(r_p)=X^TR=x_1R_1+x_2R_2+\dots+x_nR_n$; Minimize risk $Min\sigma_p^2=X^T\Sigma X=[x_1,x_2,\dots,x_n]\Sigma[x_1,x_2,\dots,x_n]^T$; Constraint condition: $x_1+x_2+\dots+x_n=1$. Among them, $E(r_p)$ is the comprehensive expected return rate of the investment portfolio of three commercial real estate projects. $R_i=E(r_i)$ ($i=1,2,3$) Indicates the expected rate of return for type i commercial real estate projects; x_i is the weight of type i commercial real estate projects in the portfolio.; n Is the number of items combined ($n=3$); σ_p^2 is the variance of three kinds of commercial real estate projects. Σ Is the covariance matrix of n items, This article $\Sigma=[\sigma_{11},\sigma_{12},\sigma_{13};\sigma_{21},\sigma_{22},\sigma_{23};\sigma_{31},\sigma_{32},\sigma_{33}]$

The above Markowitz model is to consider the portfolio under the overall risk, and only the system risk in the total risk of single investment has an impact on the expected return and portfolio return of the portfolio. Therefore, before seeking the optimal portfolio ratio, the system risk needs to be separated^[7]. Thus, the Markowitz optimization model with B coefficient as measure is obtained: minimizing risk $Min\sigma_p^2=X^T\Sigma X=[x_1,x_2,\dots,x_n]$; $E(r_p)=X^TR=x_1R_1+x_2R_2+\dots+x_nR_n=x_1(a_1+R_m\beta_1)+x_2(a_2+R_m\beta_2)+\dots+x_n(a_n+R_m\beta_n)$; Constraint condition: $x_1+x_2+\dots+x_n=1$; $1\geq x_i\geq 0$. among them, R_m is the average rate of return of the whole commercial real estate. R_0 is the lowest rate of return expected by investors; β_i is the nondispersible risk measurement coefficient, which reflects the risk of the investment portfolio and the variation degree of the return rate of the portfolio relative to the return rate of the whole market portfolio. The calculation formula is: $\beta_i=Cov(\sigma_i, \sigma_m)/\sigma_i^2=P_{im}(\sigma_i/\sigma_m)$ is the correlation between project i and non-residential investment, the standard deviation of non-residential industry, and σ_i is the standard deviation of project i . When $\beta_i=1$, it reflects that the risk value is equal to the risk of the market as a whole; When $\beta_i>1$, the risk value of the investment portfolio is more risky than the average level of the market as a whole. When $\beta_i<1$, it reflects that the risk value of its investment portfolio is lower than the average risk of the market as a whole.

2.2 Empirical Analysis of Markowitz Optimization Model Based on Measurement of B Coefficient

This paper adopts the sales price index of commercial real estate from July 2013 to April 2018 (see Appendix 1 for specific sample data) to conduct empirical research on REITs' portfolio optimization in commercial real estate nationwide.

2.2.1 Sample data analysis

Table 1 Monthly Return Rate-Variance Statistical Analysis of Various Commercial Real Estate

	Non-residential housing Expected rate of return (%)	Expected Return Rate of Office Building (%)	Expected Return Rate of Commercial Premises (%)	Other accommodation Expected rate of return (%)
Average expected annual revenue	4.72	6.61	3.19	4.16
Annual variance	1.7215	3.8072	1.5521	8.4591
Annual standard deviation	1.3120	1.9512	1.2458	2.9084

By analyzing the monthly sales price index of commercial real estate nationwide (see Table 1 above), we can easily find that the average monthly return rate of commercial real estate is in a stable growth state from 2013 to early 2018. Through statistical analysis of the data, we get the annual return rate of the whole commercial real estate is 4.72%, and the annual variance is 1.7215. The annual return rate of the office building is 6.61, nearly 2 percentage points higher than that of the whole commercial real estate, with an annual variance of 3.8072, and the risk is obviously higher than that of the whole commercial real estate. The annual return rate of selling business premises is 3.19, lower than that of the whole commercial real estate, but the annual variance is only 1.5521, lower than the risk level of the whole commercial real estate, which can reduce the risk of the whole investment through combination. The annual return rate of other commercial real estate projects is 4.16, which is also slightly lower than the annual return rate of the whole commercial real estate, and the annual variance reaches 8.4591, which is far higher than the risk of the whole commercial real estate. Increasing this part of investment will not only reduce the return of the whole investment portfolio, but also greatly increase the risk of the whole investment.

Therefore, in the investment portfolio, the return rate of the investment portfolio should be increased by investing in office buildings, and the risks should be reduced by investing in commercial premises. At the same time, the investment in other commercial real estate projects should be minimized.

2.2.2 β Coefficient analysis

β_i is a nondispersible risk measurement coefficient, which reflects the risk of the investment portfolio, that is, the variation degree of the return rate of different commercial real estate investment portfolios relative to the return rate of the whole market portfolio, $\beta_i = \text{Cov}(\sigma_i, \sigma_m) / \sigma_i^2 = \rho_{im} (\sigma_i / \sigma_m)$. Usually we use CAPM model (capital asset pricing model):

$E(r_i) = R_f + \beta_i(R_m - R_f)$. Among them: $E(r_i)$ is the expected rate of return for the i type of commercial real estate project; R_f is risk-free rate of return; R_m is the average rate of return for commercial real estate as a whole (see Table 2 below):

Table 2 β Coefficient Table

β_1	β_2	β_3
1.4331	0.9097	1.8292

β_1 , β_2 and β_3 represent the nondispersible risk measurement coefficients of office buildings, commercial buildings and other commercial buildings respectively. $\beta_1, \beta_3 > 1$ indicates that the portfolio risk value of office buildings and other commercial projects is more risky than the average level of the market as a whole. $\beta_2 \approx 1$ indicates that the risk value of commercial business premises is basically similar to that of the market as a whole, and the risk of the entire investment can be reduced through the portfolio.

2.2.3 Portfolio ratio analysis

According to Markowitz optimization model measured by β coefficient, we can find that when the minimum standard value of commercial real estate investment return rate is $R_0=R_m=4.72\%$ (the overall expected return rate of commercial real estate), the optimal proportion of REITs in China's commercial real estate investment portfolio is: office buildings account for about 45%, commercial business premises account for about 55%, and other commercial premises account for 0. This shows that REITs should reduce the investment in other commercial premises and try not to carry out REITs investment portfolio on other commercial premises. According to the above portfolio, the overall investment risk is $1.6977 < 1.7215$, i.e. the overall investment portfolio reduces the investment risk of commercial real estate and increases the return rate (see appendix 2 for detailed results). When the minimum standard rate of return is $R_0=5.5\%$, the optimal ratio of investment portfolio is: office building accounts for 67.36%, commercial business premises for 32.64%, other commercial premises are still 0, and the overall investment risk is 1.8723. The government and investment institutions can make the optimal investment portfolio of various REITs according to their own risk preferences.

2.2.4 Portfolio Optimization Analysis

As other commercial real estate in the investment portfolio will have a decline in the overall return on investment, with increased risks and poor investment benefits, only the investment portfolio optimization analysis of office buildings and commercial business premises will be discussed in the investment optimization analysis.

First of all, through the simulation of the portfolio under Markowitz model, we can obtain the return rate and standard deviation of the portfolio under different combinations of office buildings and commercial buildings in the country. In the simulation, we find that: with the increase of the investment proportion of office buildings, the return rate and standard deviation of the portfolio are also increased, and tend to increase linearly. The return rate and standard deviation of the portfolio are positively correlated with the proportion of office buildings. Similarly, with the increase in the proportion of commercial housing investment, the portfolio return rate shows a corresponding downward trend. Through linear regression, it is found that the portfolio return rate and standard deviation are negatively correlated with the proportion of commercial housing (see Table 3 below and Appendix 2 for details):

Table 3 Markowitz Model Return-Standard Deviation Table for Various Portfolio

Proportion of investment in commercial premises	Office building Investment ratio	Combination Rate of return	Combination Standard deviation
100.00%	0%	3.2	1.24
99.5%	0.5%	3.2171	1.238024
99.0%	1.0%	3.2342	1.23611
M	M	M	M
77.0%	23.0%	3.9866	1.216993
76.5%	23.5%	4.0037	1.218061
76.0%	24.0%	4.0208	1.219196
M	M	M	M
1.0%	99.0%	6.5858	1.935864
0.5%	99.5%	6.6029	1.942924
0%	100.0%	6.62	1.95

It can be seen that with the change of investment portfolio, the rate of return changes more than the standard deviation. The simulation calculation also shows that the national investment institution and the government management institution can adjust the investment proportion of REITs in commercial real estate according to their own risk preferences, increase the investment

proportion of office buildings to improve the return rate of the overall investment portfolio, and increase the investment proportion of commercial business premises to reduce risks.

3. Conclusion

In this paper, Markowitz optimization model with B coefficient as measurement is adopted. Through analyzing the sales price index of commercial real estate from July 2013 to April 2018, an empirical study is conducted on the optimization of REITs' investment portfolio in commercial real estate nationwide. The optimal investment portfolio with a minimum return rate of 4.62% for commercial real estate projects in China is found: office buildings account for 4.5%, commercial business premises account for 5.5%, and other commercial premises account for 0, which is in line with the actual situation. This paper provides an empirical basis for a detailed understanding of REITs' optimal portfolio in commercial real estate, which is conducive to the government to effectively control the investment proportion of REITs in commercial real estate projects according to China's investment risk preference, thus promoting the healthy and orderly development of commercial real estate in China. Various investment institutions can also optimize their portfolios according to the model when investing in commercial real estate REITs.

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Appendix 1

	Non-domestic building Sales price index	Non-domestic building Expected rate of return(%)	Office sales price index	Expected rate of return of office building(%)	Commercial occupancy sales price index	Expected rate of return on commercial occupancy(%)	Other housing sales price index	Other occupancy Expected rate of return(%)
July 2005	100.4	0.4	100.4	0.4	100.35	0.35	100.1	0.1
August 2005	101.4	1.4	103.0	3.0	99.9	-0.1	100.2	0.2
September 2005	100.1	0.1	99.8	-0.2	99.95	-0.05	101.3	1.3
M	M	M	M	M	M	M	M	M
October 2008	99.8	-0.2	99.9	-0.1	99.6	-0.4	99.8	-0.2
November 2008	99.2	-0.8	98.9	-1.1	99.4	-0.6	99.5	-0.5
M	M	M	M	M	M	M	M	M
December 2009	100.5	0.5	100.6	0.6	100.5824	0.5824	100.3494	0.3494
January 2010	100.5	0.5	100.7	0.7	100.3	0.3	100.2	0.2
February 2010	100.7	0.7	101.3	1.3	100.4	0.4	100.1	0.1
March 2010	100.5	0.5	100.6	0.6	100.5	0.5	100.1	0.1
April 2010	101.0	1.0	101.7	1.7	100.6	0.6	100.3	0.3
Monthly mean		0.39		0.55		0.26		0.34
Annual average expected earnings		4.72		6.61		3.19		4.16
Monthly variance		0.1434		0.3172		0.1293		0.7049
Annual variance		1.7215		3.8072		1.5521		8.4591
Monthly standard deviation		0.3787		0.5632		0.3596		0.8396
Annual standard deviation		1.3120		1.9512		1.2458		2.9084
Covariance (non-residential vs. office)		0.2056						
Covariance (non-residential vs. other)		0.2624						
Covariance (non-residential vs. commercial)		0.1305						
Covariance (other with office building)		0.3577						
Covariance (Commercial vs. Office)		0.1744						
Covariance (other and business)		0.2222						

Appendix 2

Office building Investment ratio(%)	assembly yield	assembly Standard deviation	Office building Investment ratio(%)	assembly Yield(%)	assembly Standard deviation (%)	Office building Investment ratio	assembly Yield	assembly Standard deviation (%)	Office building Investment ratio	assembly Yield(%)	assembly Standard deviation (%)	Office building Investment ratio	assembly Yield(%)	assembly Standard deviation (%)
0%	3.2	1.24	20.50%	3.9011	1.212644	41%	4.5851	1.291958	60.50%	5.2691	1.458992	81%	5.9702	1.694214
0.50%	3.2171	1.238024	21%	3.9182	1.21338	41.00%	4.6022	1.29517	61%	5.2862	1.464065	81.50%	5.9873	1.700551
1%	3.2342	1.23611	21.50%	3.9353	1.214184	42%	4.6193	1.298436	61.50%	5.3033	1.469177	82%	6.0044	1.706913
1.50%	3.2513	1.23426	22%	3.9524	1.215054	42.00%	4.6364	1.301757	62%	5.3204	1.474326	83%	6.0215	1.713298
2%	3.2684	1.232473	23%	3.9695	1.21599	43%	4.6535	1.305132	62.50%	5.3375	1.479512	83.00%	6.0386	1.719707
2.50%	3.2855	1.23075	23.00%	3.9866	1.216993	43.00%	4.6706	1.30856	63%	5.3546	1.484734	84%	6.0557	1.72614
3%	3.3026	1.22909	24%	4.0037	1.218061	44%	4.6877	1.312041	63.50%	5.3717	1.489994	84.00%	6.0728	1.732596
3.50%	3.3197	1.227495	24.00%	4.0208	1.219196	44.00%	4.7048	1.315576	64%	5.3888	1.495289	85%	6.0899	1.739075
4%	3.3368	1.225964	25%	4.0379	1.220396	45%	4.7219	1.319162	64.50%	5.4059	1.50062	85.00%	6.107	1.745576
4.50%	3.3539	1.224498	25.00%	4.055	1.221662	45%	4.739	1.322801	65%	5.423	1.505986	86%	6.1241	1.7521
5%	3.371	1.223097	26%	4.0721	1.222994	45.50%	4.7561	1.326491	65.50%	5.4401	1.511387	86.00%	6.1412	1.758646
5.50%	3.3881	1.221761	26.00%	4.0892	1.22439	46%	4.7732	1.330232	66%	5.4572	1.516823	87%	6.1583	1.765214
6%	3.4052	1.22049	27%	4.1063	1.225851	46.50%	4.7903	1.334023	66.50%	5.4743	1.522292	87.00%	6.1754	1.771803
6.50%	3.4223	1.219284	27.00%	4.1234	1.227377	47%	4.8074	1.337865	67%	5.4914	1.527796	88%	6.1925	1.778414
7%	3.4394	1.218145	28%	4.1405	1.228968	47.50%	4.8245	1.341756	68%	5.5085	1.533333	88.00%	6.2096	1.785046
8%	3.4565	1.217071	28.00%	4.1576	1.230622	48%	4.8416	1.345697	68.00%	5.5256	1.538903	89%	6.2267	1.791699
12%	3.5933	1.210873	32%	4.2944	1.246137	52%	4.9784	1.378948	72.00%	5.6624	1.584609	92.50%	6.3635	1.845651
M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
15%	3.6959	1.209039	35%	4.397	1.260367	55.00%	5.081	1.405806	75%	5.765	1.620157	95.50%	6.4661	1.886917
15%	3.713	1.208969	35.50%	4.4141	1.262949	56%	5.0981	1.410435	75.50%	5.7821	1.626182	96%	6.4832	1.893857
17.50%	3.7985	1.20963	38.00%	4.4996	1.276738	58.00%	5.1836	1.434209	78%	5.8676	1.656713	99%	6.5687	1.928821
18%	3.8156	1.209965	39%	4.5167	1.279669	59%	5.2007	1.439086	78.50%	5.8847	1.6629	99.00%	6.5858	1.935864
18.50%	3.8327	1.210366	39.00%	4.5338	1.282657	59.00%	5.2178	1.444004	79%	5.9018	1.669112	100%	6.6029	1.942924
19%	3.8498	1.210835	40%	4.5509	1.285702	60%	5.2349	1.448961	79.50%	5.9189	1.67535	100.00%	6.62	1.95